

The Beauty of Anisotropic AMR: Omnitrees for Efficient Dyadic Discretizations

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Plasma microturbulence

screenshot from <http://www.iter.org/machine>

Solve a higher-dimensional 6(+1)-d system for the distribution function $F(x_1, x_2, x_3, v_1, v_2, v_3, t)$

Important: areas with **fine / thin features**

Octrees: state-of-the-art for structured AMR

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Problem: isotropic refinement
⇒ curse of dimensionality!

plasma microturbulence ⚡ octree AMR

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100 010 001

110 011 101

Z order is still applicable!

By David Epstein, based on a image by Hesperian. - Own work, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=3879675>

Novel Omnitrees: allow all splits

Linearized storage:

101 000 001 000 000 010 100 000 000 000 101 000 000 010 000 101 000 000 000 000 000

0 0 0 1 1 0 1 1 0 1 1 0 0 0 1

tree descriptor + function values

omnitrees: structured AMR for anisotropy

shape approximation on 4,166 3d objects

1 inside
0 outside

L_1 error (Monte Carlo)

Allowed number of cuboids in $\Omega = (0, 1)^3$

Legend: Octree (orange squares), Omnitree (teal diamonds)

Median L_1 error (Monte Carlo)

Median storage size (tree and data vector) [bit]

5x less memory for similar error

Legend: Octree (orange squares), Omnitree (teal diamonds)

increased convergence order ⇒ lower memory footprint

Dimension-independent Convergence Rate

$$r = \frac{\log\left(\frac{e_1}{e_2}\right)}{\log\left(\frac{N_2}{N_1}\right)}$$

with e_1, e_2 : errors at different resolutions,
 N_1, N_2 : their number of cuboids

$N_2^{\text{ct}} = 2^d N_1, N_2^{\text{omni}} = 2N_1, e_2^{\text{ct}} \leq e_2^{\text{omni}}$

⇒ $1 \leq \frac{r_{\text{omni}}}{r_{\text{ct}}} \leq \frac{\log(2^d)}{\log(2)} = d$

convergence rates r

	octree	omnitree
r_{mean}	0.39	0.59
r_{median}	0.56	0.76

⇒ increase by 1.5x

shape approximation	plasma microturbulence
moderately anisotropic	very anisotropic
2 – 4d	4 – 7d
single bits	usually double

arxiv.org/abs/2508.06316

Preprint with animations!

github.com/TheresaPollinger/OmnitreeDyada

try the code!
pip install dyada

should fit plasma microturbulence even better